

The Influence of Servo Valve Dynamic Characteristics on the Closed Loop Control

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1 Introduction

The following observations are intended to extend understanding of the various relationships in closed loop control circuits and to facilitate accurate assessment of the properties of a control system.

Simple rules of thumb will be given instead of complicated mathematical calculations.

2 Closed Loop Positional Control Circuit

Determination of effective loop gain K_{vopt} and its influence on closed loop control.

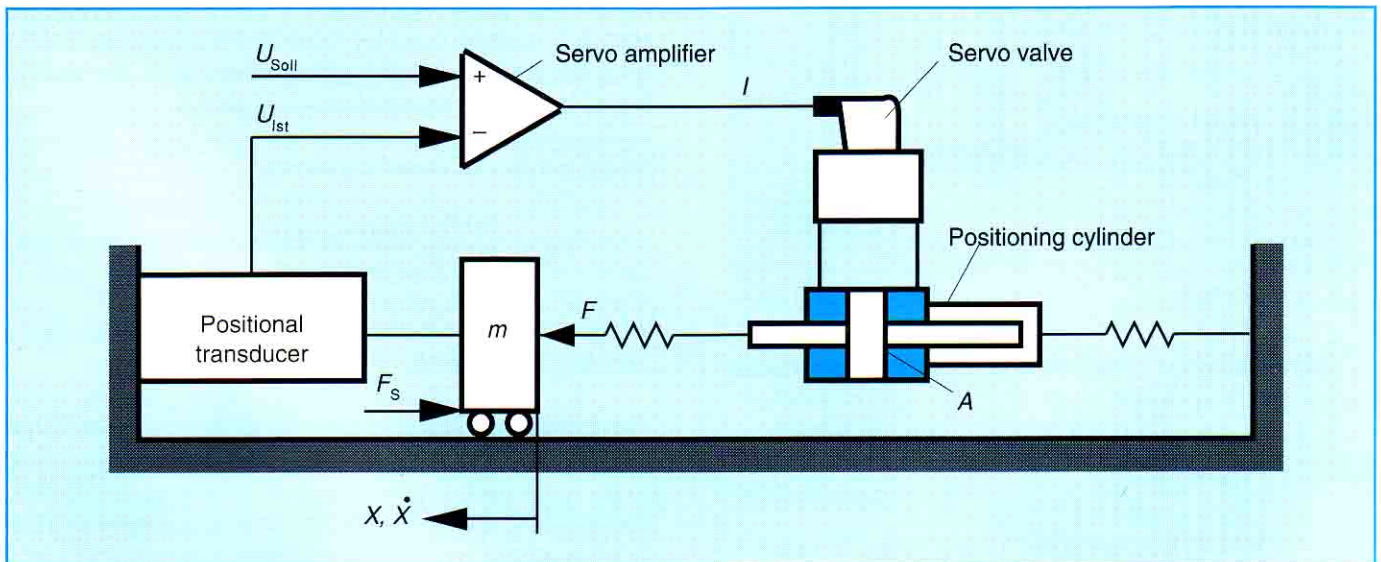


Fig. 246

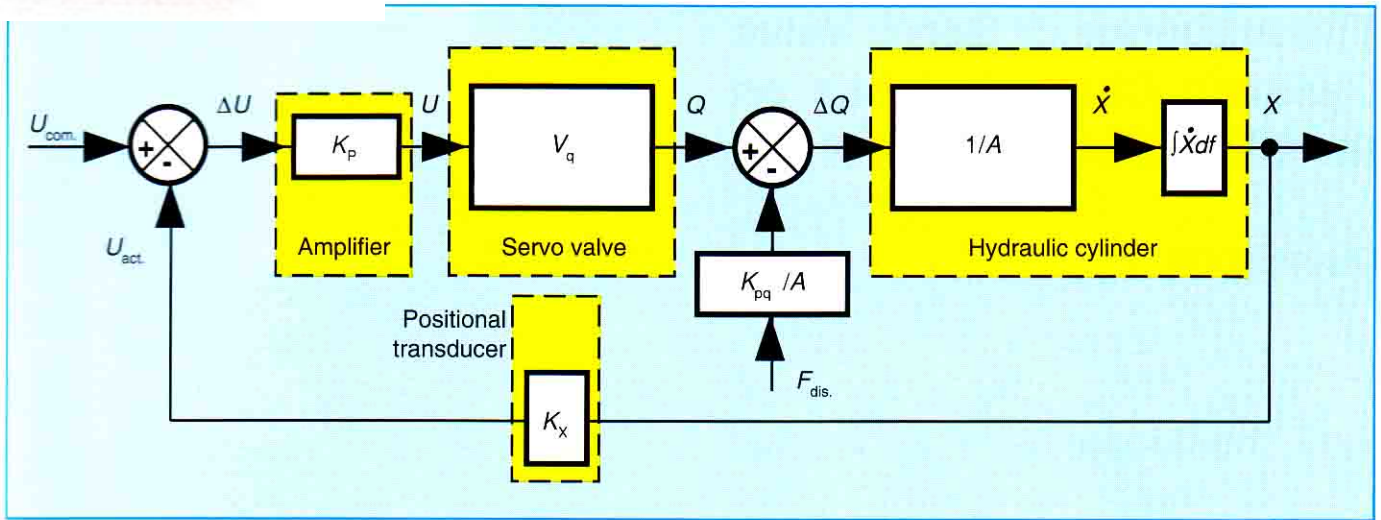


Fig. 247: Simplified block diagram

Disregarding the dynamic properties of valves and servo cylinders, a first order system is obtained as follows:

The loop gain K_V is equal to the product of transfer element gain factors in the closed loop control circuits.

$$K_V = \frac{K_p \cdot V_q \cdot K_x}{A} \text{ in } s^{-1}$$

- V_q = Flow gain in $cm^3/s/V$
- V_p = Pressure gain in $bar/Volt$
- K_p = Electrical gain
- K_x = Positional transducer gain in V/cm
- K_{pq} = Pressure-flow gain (V_q/V_p) in $cm^3/s/bar$
- A = Cylinder area in cm^2
- K_V = Loop gain in s^{-1}

2.1 Time Constant for Closed Loop Control

The time constant is proportional to $1/K_V$

$$T = \frac{1}{K_V} \text{ in } s$$

i.e., the greater the loop gain K_V the faster the system is.

2.2 Rigidity

When stationary, the rigidity with respect to external force disturbances is given by

$$C = \frac{F_{dis}}{X} = \frac{K_V \cdot A^2}{K_{pq}}$$

Hence rigidity is proportional to the loop gain and inversely proportional to pressure-flow gain K_{pq} .

$$K_{pq} = \frac{V_q}{V_p} \text{ in } \frac{cm^3/s}{bar}$$

- V_q = Flow gain in $cm^3/s/V$
- V_p = Pressure gain in bar/V

2.3 Positional Error

Normally, less than 5 % of the valve current is required in closed loop positional control to change the velocity to zero or to compensate for external disturbances. This is because the full system pressure is available at max. 5 % signal to carry out corrections.

Hence positional error is given by

$$\Delta X \leq \frac{0,05 \cdot v_{max}}{K_V} \text{ in } mm$$

v_{max} is the velocity which would be present at 100 % opening of the servo valve.

Therefore, it is clear that the loop gain must be as large as possible.

The larger K_V , the better the positioning accuracy and the more rigid the system (with respect to external force disturbances).

Furthermore, it may be seen that the nominal flow of the servo valve $Q = A \cdot V_{max}$ should be selected as small as possible.

For stability reasons, loop gain cannot be selected above a certain value.

If loop gain K_v is greater than critical circuit frequency $K_v \text{ crit}$, the system will oscillate when disturbed, i.e. the system will be unstable.

3 What is the Maximum Value of K_v ?

Two cases exist as follows:

3.1 The servo valve frequency ω_v (frequency at -90° phase offset) is considerably higher than the natural frequency of the load ω_L ($\omega_v \geq 3 \cdot \omega_L$).

In this case, the dynamics of the part of the system with the higher natural frequency may be neglected. Hence closed loop control is reduced to a 3rd order system given by

$$K_v < K_{v \text{ crit}} = 2 \cdot D \cdot \omega_L$$

D = dimensionless damping factor

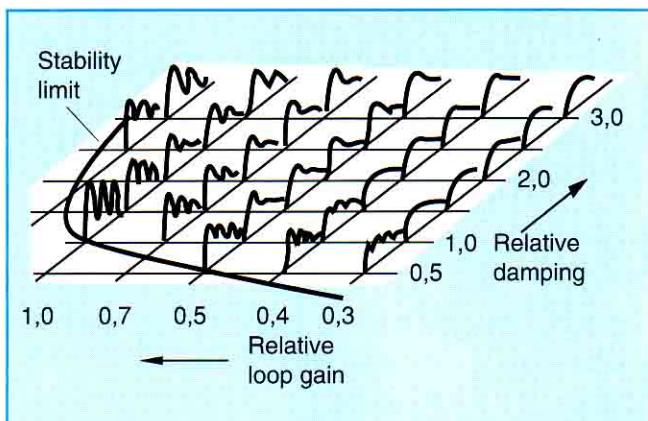
i.e. open loop gain K_v must be selected so that it is smaller than $K_{v \text{ crit}}$.

Diag. 79 shows the time characteristic of such a 3rd order closed control loop system where relative damping and relative gain are the parameters. The optimum gain $K_{v \text{ opt}}$ is normally derived from this time characteristic, i.e. from the step response. If K_v is kept small at a given damping, a uniformly rising step response results. However, if K_v is extremely large, intense superimposed oscillation occurs.

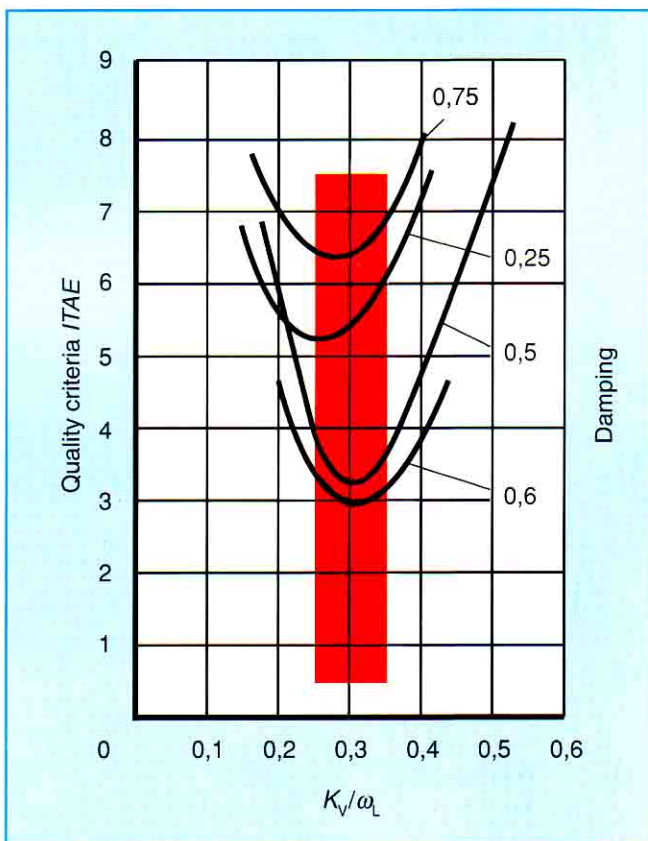
Quality criteria may be defined based on the progression of this step response (transient response). The *ITAE* criteria (Integral of Time multiplied by Absolute Error) is often used:

$$ITAE = \int_0^{\infty} t |X_E - X_A| \cdot dt$$

The optimum loop gain is that at which this *ITAE* value is at a minimum. If K_v is varied at constant damping and if *ITAE* is distributed over the relative gain K_v/ω_L , Diag. 80 is obtained.



Diag. 79: Time characteristic of 3rd order closed loop control



Diag. 80

It can be seen that for the range of typical damping coefficients ($0.2 < D < 0.9$), the optimum *ITAE* values are between $K_v/\omega_L = 0.25$ and 0.35 .

This results in **Rule 1**

$$K_{v \text{ opt}} \approx \frac{1}{3} \omega_L \text{ in s}^{-1}$$

This gain, also termed velocity gain, is the product of hydraulic gain and electrical gain.

3.2 Considering Both Natural Frequencies

This results in a 5th order system. Stability considerations result in a critical frequency ω_{crit} and a critical loop gain $K_{V_{crit}}$ which are dependent on the two natural frequencies ω_v = natural valve frequency and ω_L = natural load frequency

The critical frequency ω_{crit} is always smaller than the smaller of the two frequencies ω_v and ω_L .

Neglecting damping factors **Rule 2** is given by

$$\omega_{crit} \approx \frac{\omega_v \cdot \omega_L}{\omega_v + \omega_L} \text{ in } s^{-1}$$

Optimum loop gain is hence given by **Rule 3**:

$$K_{V_{opt}} \approx \frac{1}{3} \omega_{crit} \text{ in } s^{-1}$$

In both cases the following applies:

Accuracy of position and rigidity with respect to external disturbance forces require a high electrical gain K_p .

Hydraulic gain should however be only as large as necessary (see positioning error).

Rule 4

Use valve with the smallest possible nominal flow. Normally, this is also the valve with better dynamic characteristics.

3.3 Increasing Loop Gain

If the calculation of the drive shows that accuracy requirements will not be met, loop gain may be increased by an appropriate control circuit.

The following circuits enable optimum loop gain to be increased and hence lead to an improvement in positioning accuracy.

- PD regulator circuit
- Feedback of load pressure and
- Feedback of velocity
- An integral circuit may infinitely increase the accuracy, but dynamic requirements limit the I-component.
- An increase in gain and hence in damping is achieved by means of a by-pass leakage between the actuator connections. However, static rigidity is reduced as a result.

- A very effective measure which allows a higher loop gain is achieved by means of an additional feedback of acceleration in a multi-loop closed loop control system.

4 Determination of Nominal Frequencies

4.1 Servo Valve

The frequency response of the servo valve is obtained from the frequency response curve.

Frequency at -90° phase

4.2 Hydraulic cylinder

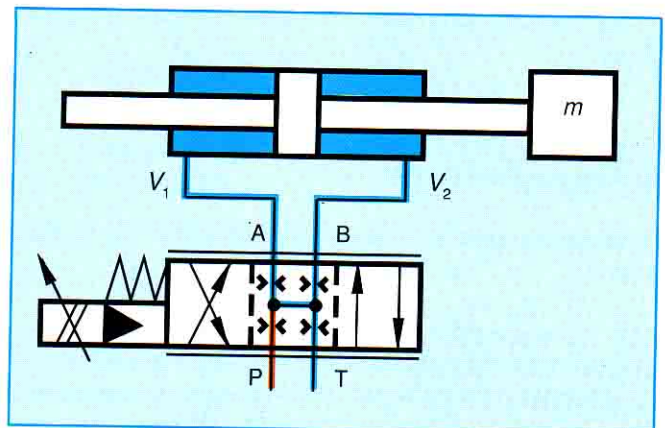


Fig. 248: Determination of natural frequency with double rod cylinder

Double rod cylinder

$$\omega_L = \sqrt{\frac{2 \cdot E_{oil} \cdot A_R^2}{V \cdot m_{ref}}} \text{ in } s^{-1}$$

$$V = V_1 = V_2 = \frac{A_R \cdot H}{2} + V_{LR} \text{ in } cm^3$$

- E_{oil} = Modulus of elasticity of oil $1,4 \cdot 10^7$ in $kg/cm \cdot s^2$
- A_R = Annulus area of cylinder in cm^2
- H = Cylinder stroke in cm
- V = Total oil volume in cm^3
- m_{ref} = Reflected (or equivalent mass) = m/i^2 (with respect to cylinder axis) in kg
- i = Transfer ratio
- V_{LR} = Pipe volume on annulus side of cylinder in cm^3

The minimum natural frequency occurs at the centre of the cylinder stroke ($h_k = H/2$).

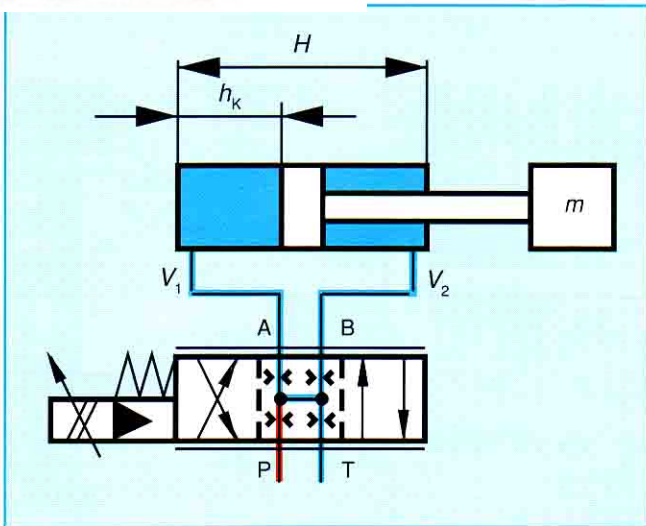


Fig. 248a: Determination of natural frequency with single rod cylinder

$$\omega_L = \sqrt{\frac{C_1}{m_{\text{refl}}} + \frac{C_2}{m_{\text{refl}}}} \text{ in s}^{-1}$$

$$\omega_L = \sqrt{\frac{E_{\text{oil}} \cdot A_K^2}{V_1 \cdot m_{\text{refl}}} + \frac{E_{\text{oil}} \cdot A_R^2}{V_2 \cdot m_{\text{refl}}}} \text{ in s}^{-1}$$

- E_{oil} = Modulus of elasticity of oil $1,4 \cdot 10^7$ in $\text{kg/cm} \cdot \text{s}^2$
- A_R = Annulus area of cylinder in cm^2
- A_K = Piston area of cylinder in cm^2
- V_1 = Oil volume on piston side in cm^3
- V_2 = Oil volume on annulus side in cm^3
- m_{refl} = Reflected (or equivalent mass) = m/i^2 (with respect to cylinder axis) in kg
- i = Transfer ratio
- H = Stroke in cm
- h_K = Cylinder stroke at min natural frequency in cm
- V_{LK} = Pipe volume on piston side in cm^3
- V_{LR} = Pipe volume on annulus side in cm^3
- $V_1 = A_K \cdot h_K + V_{LK}$ in cm^3
- $V_2 = A_R \cdot (H - h_K) + V_{LR}$ in cm^3

$$h_K = \frac{\left(\frac{A_R \cdot H}{\sqrt{A_R^3}} + \frac{V_{LR}}{\sqrt{A_R^3}} - \frac{V_{LK}}{\sqrt{A_K^3}} \right)}{\left(\frac{1}{\sqrt{A_R}} + \frac{1}{\sqrt{A_K}} \right)} \text{ in cm}$$

The minimum natural frequency occurs at cylinder position h_K .

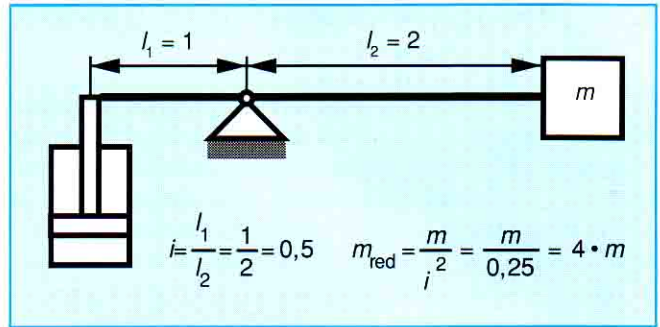


Fig. 249: Example of reflected mass

4.3 Hydraulic motor

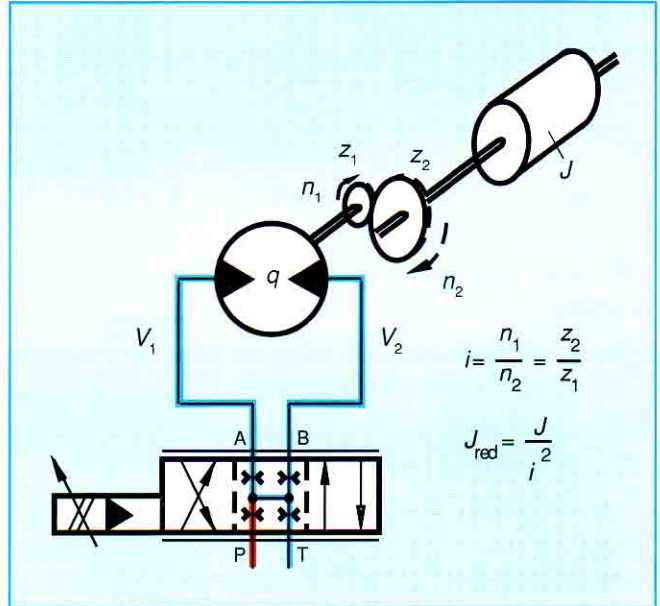


Fig. 250: Determination of natural frequency of a motor drive

$$\omega_L = \sqrt{\frac{2 \cdot \left(\frac{q}{2 \cdot \pi} \right)^2 \cdot E_{\text{oil}}}{V_1 \cdot J_{\text{refl}}}} \text{ in s}^{-1}$$

- E_{oil} = Modulus of elasticity of oil $1,4 \cdot 10^7$ in $\text{kg/cm} \cdot \text{s}^2$
- q = Flow in cm^3
- V_1 = Oil volume on one side = $q/2 + V_{LR}$ in cm^3
- J_{refl} = Reflected (or equivalent) moment of inertia = J/i^2 ($i = n_1/n_2 = z_2/z_1 \dots$) with respect to motor shaft in kgcm^2

5 Selection of Measuring System

As already mentioned, a measurement system is required for the control of a physical variable. The system must be able to convert the relevant variable into an electrical signal (current or voltage). Hence, devices are required which measure distances, angles, velocities, speeds, pressures, forces, torques and accelerations. A number of measurement techniques are available for each of these variables. The choice of which technique to use is dependent on measurement range, accuracy requirement, service life, ambient conditions etc. The variety of measurement devices available is so large, that it is only possible to provide a very general overview.

The following is generally applicable:

- Closed loop control is only as accurate as the measurements carried out.
- Measurement systems are characterized by their transfer factors. This factor is the ratio of output voltage or current to measured variable.
- Accuracy of the measurement system must be at least 5 times greater than that required by closed loop control.
- Measurement systems must be able to measure changing variables without delay.
- Transfer factors and null points must remain constant for all operating conditions.
- Electrical signals must be processed in such a way, that they are free of or can be kept free of interference caused by neighbouring high power elements.
- Coupling of measurement system to drive must be extremely rigid and free of play.
- The measurement system must be arranged so that control variables are measured directly and no errors in measurement are produced due to secondary effects.

These few points clearly illustrate the significance of measurement systems in closed loop control technology and in servo hydraulics.

6 Calculation example

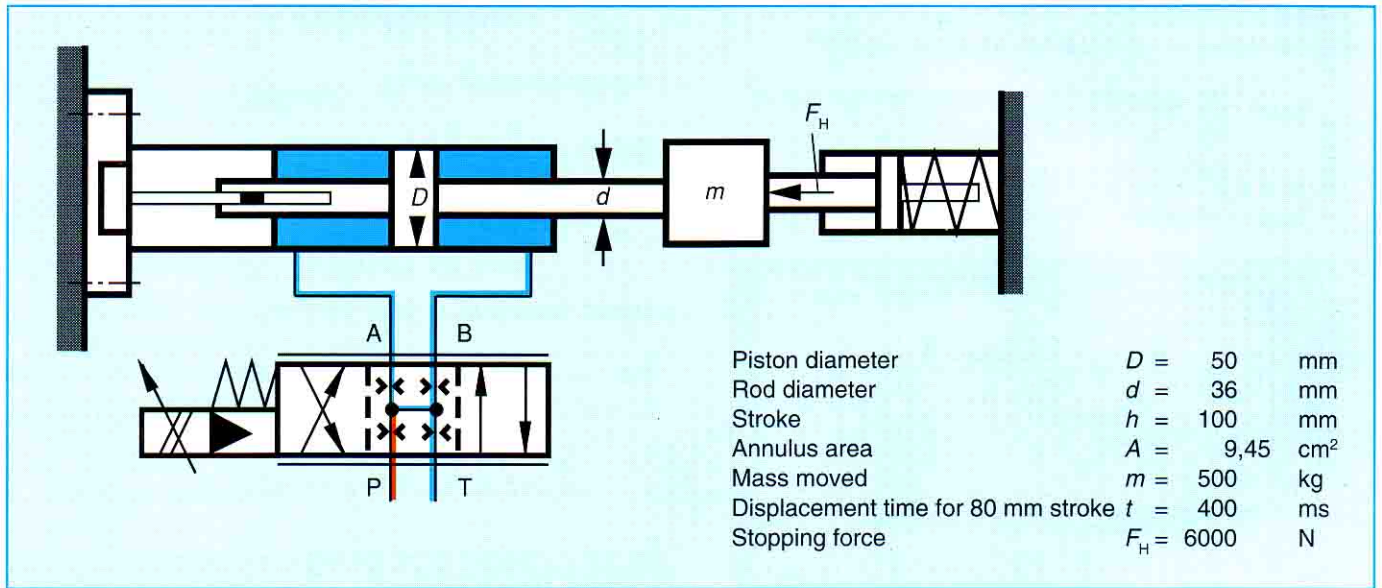


Fig. 251

6.1 Natural Frequency of Hydraulic Cylinder Mass System

Double Rod Cylinder

$$\omega_L = \sqrt{\frac{2 \cdot E_{oil} \cdot A_R^2}{V \cdot m_{refl}}}$$

If the valve is mounted directly on the cylinder, the volume is given by

$$V = \frac{H}{2} \cdot A_R$$

If this is used in the above formula for ω_0 this results in:

$$\omega_L = \sqrt{\frac{4 \cdot E_{oil} \cdot A_R}{H \cdot m_{refl}}}$$

$$\omega_L = \sqrt{\frac{4 \cdot 1,4 \cdot 10^7 \text{ (kg/cm s}^2\text{)} \cdot 9,45 \text{ (cm}^2\text{)}}{10 \text{ (cm)} \cdot 500 \text{ (kg)}}$$

$$\omega_L = 325 \text{ s}^{-1}$$

$$f_L = \frac{\omega_L}{2\pi} = 51 \text{ Hz}$$

If the natural valve frequency is considerably higher than the natural frequency of the cylinder/mass system, the loop gain K_V is given by

$$K_V < K_{Vcrit} = 2 \cdot D \cdot \omega_L \text{ (see page 205, paragraph 3.1)}$$

Rule1

$$K_{Vopt} \approx \frac{1}{3} \omega_L$$

$$K_{Vopt} \approx \frac{325}{3} = 108 \text{ s}^{-1}$$

Time Constant

$$T = \frac{1}{K_{Vopt}} = \frac{1}{108} = 0,0092 \text{ s}$$

Possible Acceleration Time

The following minimum acceleration time used in practical applications may be used:

$$T_B \approx 5 \text{ to } 6 \cdot T$$

i. e. $T_B \approx 5 \cdot T \approx 50 \text{ ms}$.

Time Constant

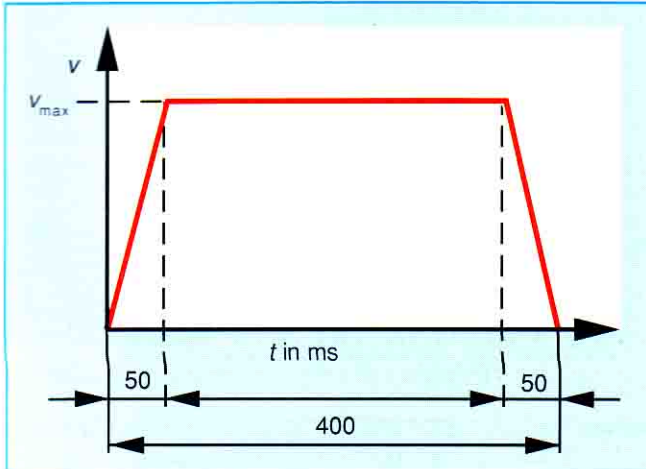
6.2 Selection of Servo Valve

Maximum Velocity

(when acceleration and deceleration are equal)

$$v_{\max} = \frac{s}{T_{\text{tot}} - T_B} = \frac{80}{0,4 - 0,050}$$

$$v_{\max} = 228 \text{ mm/s}$$



Diag. 81

Required Flow

$$Q = A \cdot v = 9,45 \text{ cm}^2 \cdot 22,8 \text{ cm/s} = 215,5 \text{ cm}^3/\text{s}$$

$$Q = 13 \text{ L/min}$$

Selection:

Servo valve with $Q_N = 20 \text{ L/min}$ at $\Delta p = 70 \text{ bar}$.

6.3 Calculation of Loop Gain Taking into Consideration Natural Valve Frequency

Damping factors will be disregarded.

Rule 2

$$\omega_{\text{crit}} \approx \frac{\omega_V \cdot \omega_L}{\omega_V + \omega_L} \text{ in } \text{s}^{-1}$$

Determination of ω_V from the Frequency Response

For a valve with $Q_N \leq 30 \text{ L/min}$, for a signal of 25% nominal flow and for 90° phase shift:

$$f_{,90^\circ} = 85 \text{ Hz at } 140 \text{ bar pilot pressure}$$

$$\omega_V = f_V \cdot 2 \pi = 85 \cdot 2 \pi = 534 \text{ s}^{-1}$$

Hence

$$\omega_{\text{crit}} \approx \frac{\omega_V \cdot \omega_L}{\omega_V + \omega_L} \approx \frac{534 \cdot 325}{534 + 325} \approx 202 \text{ s}^{-1}$$

Rule 3

$$K_{V \text{ opt}} \approx \frac{1}{3} \omega_{\text{crit}} \approx \frac{202}{3} \approx 67 \text{ s}^{-1}$$

Comparison of the two calculated loop gains (108 s^{-1} and 67 s^{-1}) shows that, the valve greatly influences the possible loop gain and must, therefore, be taken into consideration.

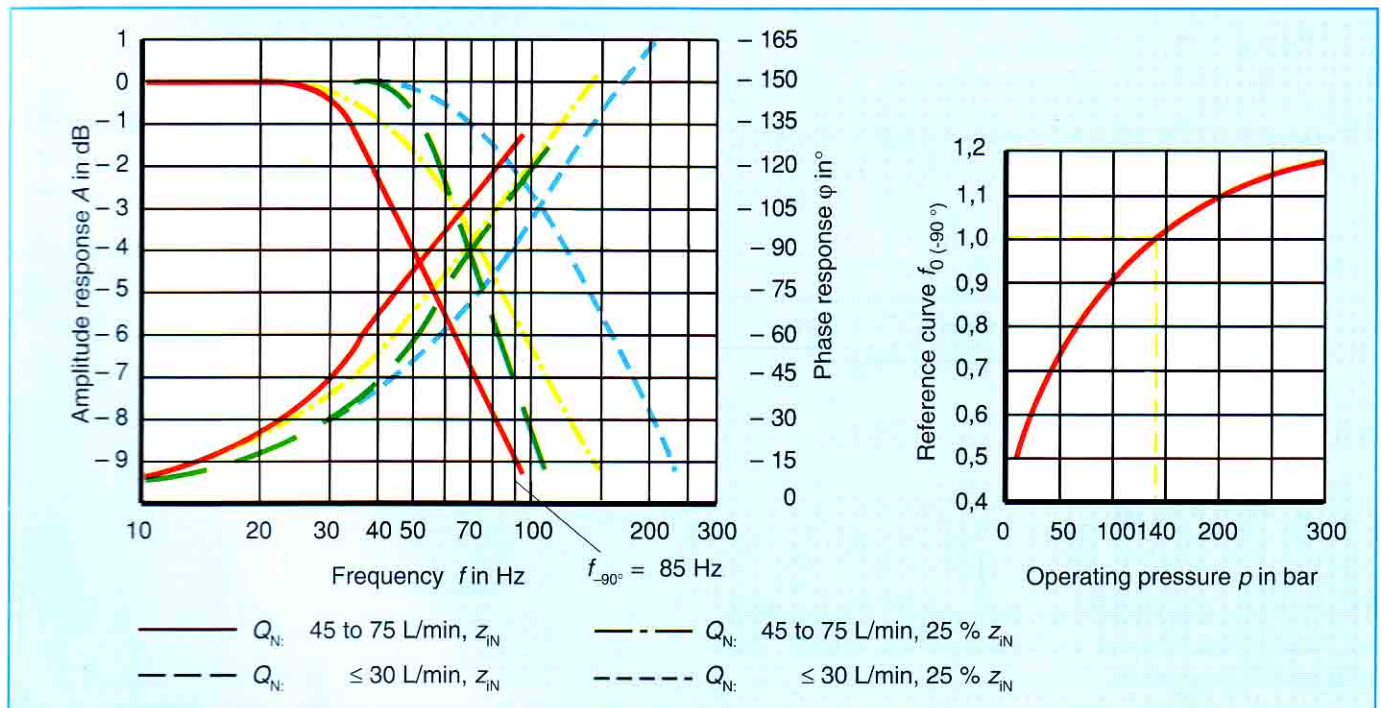


Fig. 82: Typical frequency response curves (left and dependence of frequency on operating pressure (right) for servo valves with mechanical feedback

Time Constant

$$T = \frac{1}{K_V} = \frac{1}{67} \frac{1}{s} = 0,015 \text{ s}$$

Possible Acceleration Time

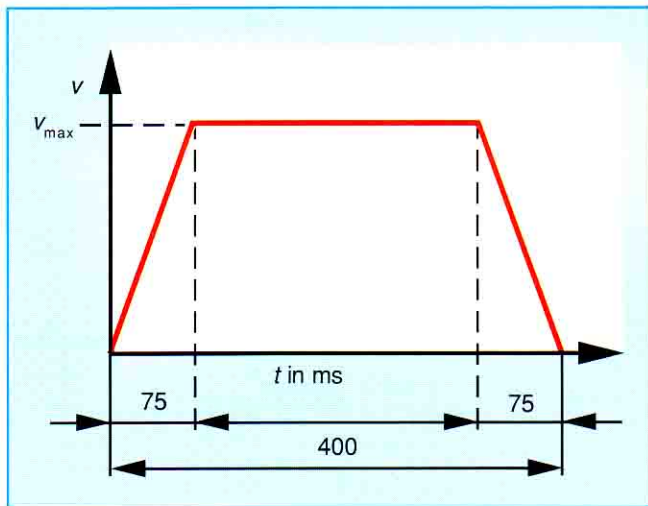
$$T_B = 5 \cdot T = 0,075 \text{ s} = 75 \text{ ms}$$

6.4 Selection of Servo Valve

Maximum Velocity

$$v_{\max} = \frac{s}{T_{\text{tot}} - T_B} = \frac{80}{0,4 - 0,075}$$

$$v_{\max} = 246 \text{ mm/s}$$



Diag. 83

Required Flow

$$Q = A \cdot v = 9,45 \text{ cm}^2 \cdot 24,6 \text{ cm/s} = 232,5 \text{ cm}^3/\text{s}$$

$$Q = 13,9 \text{ L/min}$$

Selection:

Servo valve with $Q_N = 20 \text{ L/min}$ at $\Delta p_N = 70 \text{ bar}$

Whereby $Q = Q_N \cdot \sqrt{\frac{\Delta p}{\Delta p_N}}$

Pressure Drop at Valve

$$\Delta p = \left(\frac{Q}{Q_N}\right)^2 \cdot \Delta p_N = \left(\frac{14}{20}\right)^2 \cdot 10 = 34 \text{ bar}$$

Acceleration

$$a_{\max} = \frac{v_{\max}}{T_B} = \frac{0,25 \text{ m/s}}{0,075 \text{ s}} = 3,3 \text{ m/s}^2$$

Required Force for Acceleration

$$F_B = m \cdot a_{\max} = 500 \text{ kg} \cdot 3,3 \text{ m/s}^2 = 1650 \text{ N}$$

Required Pressure for Acceleration

$$p_{B \max} = \frac{F_B}{A_R} = \frac{1650 \text{ N}}{9,45 \text{ cm}^2} = 17,4 \frac{\text{daN}}{\text{cm}^2} \triangleq 17,4 \text{ bar}$$

Pressure Requirements for Stopping Force

$$p_H = \frac{F_H}{A_R} = \frac{6000 \text{ N}}{9,45 \text{ cm}^2} = 64 \frac{\text{daN}}{\text{cm}^2} \triangleq 64 \text{ bar}$$

6.5 Calculation of System Pressure

(see Chapter: "Design Criteria for Open Loop Control with Proportional Valves")

The following is valid for acceleration:

$$p_P = \frac{2 \cdot m \cdot v}{T_B \cdot 10 \cdot A_R} + \Delta p_V + \frac{F_H}{10 \cdot A_R}$$

$$p_P = \frac{2 \cdot 500 \text{ kg} \cdot 0,25 \text{ m/s}}{0,075 \text{ s} \cdot 10 \cdot 9,45 \text{ cm}^2} + 10 \text{ bar} + \frac{6000 \text{ N}}{10 \cdot 9,45 \text{ cm}^2}$$

$$p_P = 109 \text{ bar}$$

selected

$$p_P = 110 \text{ bar}$$

6.6 Determination of Positioning Accuracy

Loop gain

$$K_V = K_p \cdot V_q \cdot \frac{1}{A} \cdot K_X = 67 \text{ s}^{-1}$$

K_p = Electrical gain	(dimensionless)
V_q = 20 L/min/10 V	= 33 cm ³ /s/V
$1/A$ = 1/9,45 cm ²	= 0,106 1/cm ²
K_X = 10 V/10 cm	= 1 V/cm

Calculation of electrical gain K_p

$$K_p = \frac{K_V}{V_q \cdot \frac{1}{A} \cdot K_X} = \frac{67 \text{ s}^{-1}}{33 \frac{\text{cm}^3}{\text{s} \cdot \text{V}} \cdot 0,106 \frac{1}{\text{cm}^2} \cdot 1 \frac{\text{V}}{\text{cm}}}$$

$K_p = 19$

Feedback Error

$$s_N = \frac{v_{\max}}{K_V} \text{ in mm}$$

v_{\max} is the maximum possible velocity when the valve is open.

$$s_N = \frac{250 \frac{\text{mm}}{\text{s}}}{67 \text{ s}^{-1}} = 3,7 \text{ mm}$$

Positioning Accuracy

Usually the positioning accuracy is better than 5% of s_N

$\Delta X \leq 0,19 \text{ mm}$

Error resulting from servo valve reversal error

How large must the closed loop error be for the servo valve to overcome its reversal error?

Assumption: reversal error = 0.2 % nominal signal

$$K_U = 0,002 \cdot 10 \text{ V} = 0,02 \text{ V}$$

Condition

$$\Delta X \cdot K_X \cdot K_p = K_U$$

$$\Delta X = \frac{K_U}{K_X \cdot K_p} = \frac{0,02 \text{ V}}{1 \frac{\text{V}}{\text{cm}} \cdot 19} = 0,001 \text{ cm}$$

$\Delta X = 0,01 \text{ mm}$

Error resulting from change in load

How large must the closed loop error be for the servo valve to compensate for the disturbance force, with respect to pressure gain V_p ?

$$F_H = \Delta X \cdot K_X \cdot K_p \cdot V_p \cdot A$$

$$\Delta X = \frac{F_H}{K_X \cdot K_p \cdot V_p \cdot A}$$

Determination of V_p

At 1 % signal, 80 % pressure is applied to the actuator

$$V_p = \frac{0,8 \cdot 110 \text{ bar}}{0,1 \text{ V}} = 880 \frac{\text{bar}}{\text{V}} \hat{=} 8800 \frac{\text{N}}{\text{cm}^2 \text{ V}}$$

$$\Delta X = \frac{6000 \text{ N cm}^2 \text{ V}}{1 \frac{\text{V}}{\text{cm}} \cdot 19 \cdot 8800 \frac{\text{N}}{\text{cm}^2} \cdot 9,45 \text{ cm}^2}$$

$\Delta X = 0,004 \text{ cm} = 0,04 \text{ mm}$

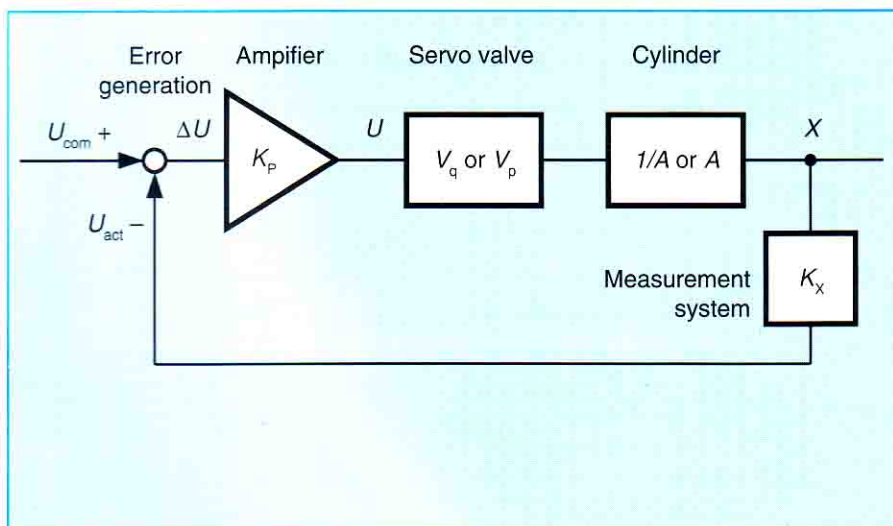


Fig. 252

Flow gain v_q and pressure gain V_p are particularly important operating curves for closed loop positional control, as they directly effect closed loop control results.

The following example clarifies the relationships.

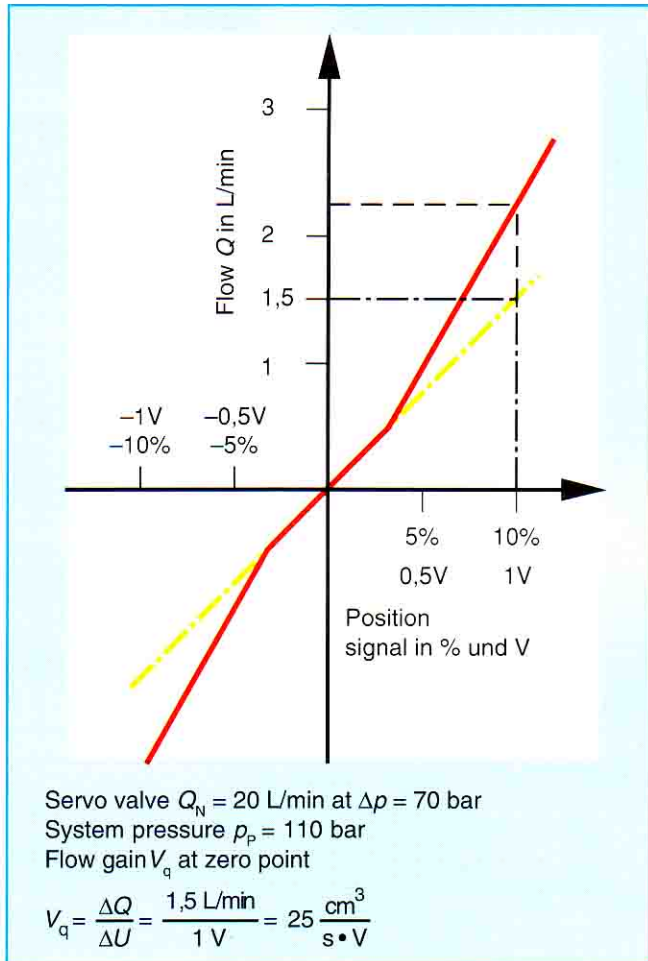


Fig. 84: Operating curve for flow with null point region greatly enlarged

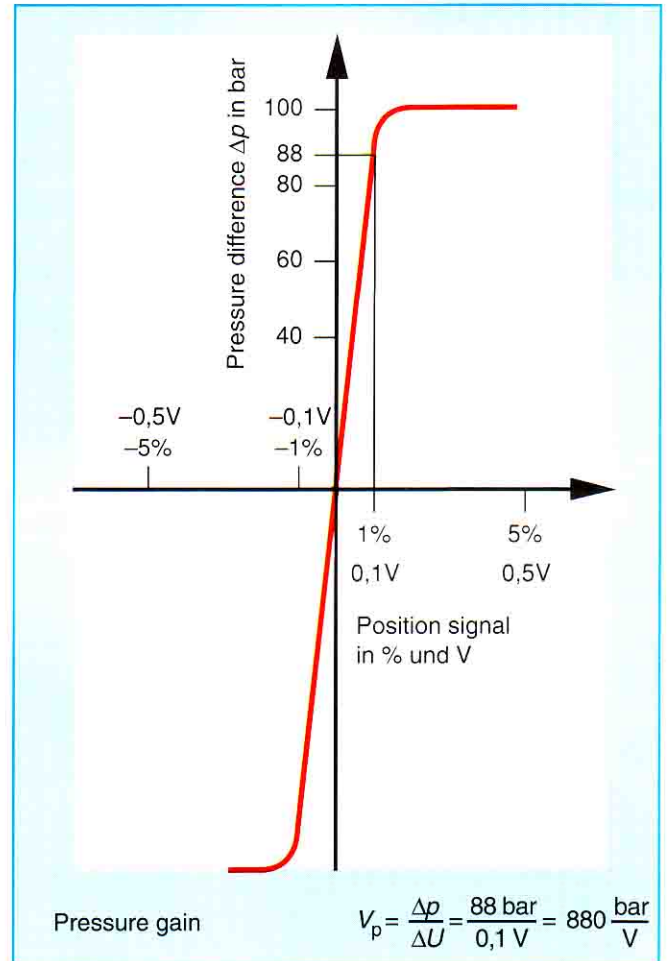


Fig. 85: Operating curve for pressure

Flow gain at zero point	$V_q = 25 \text{ cm}^3/\text{s/V}$
Pressure gain	$V_p = 880 \text{ bar/V}$
Piston area	$A = 9,45 \text{ cm}^2$
Deduced loop gain	$K_V = 67 \text{ s}^{-1}$

Hence, rigidity with respect to disturbance forces may be determined (see paragraph 2.2 on page 204).

$$\frac{F_{\text{Stör}}}{X} = \frac{K_V \cdot A^2}{K_{pq}} = K_V \cdot A^2 \cdot \frac{V_p}{V_q}$$

$$\frac{F_{\text{Stör}}}{X} = \frac{67 \text{ s}^{-1} \cdot 9,45^2 \text{ cm}^4 \cdot 880 \frac{\text{bar}}{\text{V}} \cdot 10 \frac{\text{N}}{\text{cm}^2 \text{ bar}}}{25 \frac{\text{cm}^3}{\text{s} \cdot \text{V}}}$$

$$\frac{F_{\text{Stör}}}{X} = 2,10 \cdot 10^6 \frac{\text{N}}{\text{cm}} = 2,10 \cdot 10^2 \frac{\text{N}}{0,001 \text{ mm}}$$

For an external force of e.g 6000N, the following change in position is obtained:

$$X = \frac{F_{\text{ds.}}}{2,10 \cdot 10^2 \frac{\text{N}}{\mu\text{m}}} = \frac{6000 \text{ N}}{2,10 \cdot 10^2 \frac{\text{N}}{\mu\text{m}}} \approx 29 \mu\text{m} \hat{=} 0,029 \text{ mm}$$

Hence for closed loop control:

The difference between command and actual (ΔX) positions is: $29 \mu\text{m} = 0.029 \text{ mm} = 0.0029 \text{ cm}$

The following is valid for the corrective signal to the servo valve:

$$\text{From } K_V = \frac{K_P \cdot V_q \cdot K_X}{A} \text{ it follows that } K_P \cdot K_X = \frac{K_V \cdot A}{V_q}$$

$$\dots \text{ and hence } \Delta U_{SV} = \Delta X \cdot \frac{K_V \cdot A}{V_q}$$

$$\Delta U_{SV} = \frac{0,0029 \text{ cm} \cdot 67 \text{ s}^{-1} \cdot 9,45 \text{ cm}^2 \text{ s V}}{25 \text{ cm}^3} = 0,073 \text{ V.}$$

Hence due to the pressure gain the following is obtained:

$$\Delta p = \Delta U_{SV} \cdot V_p = 0,073 \text{ V} \cdot 880 \frac{\text{bar}}{\text{V}} = 64,2 \text{ bar.}$$

This produces a force at the cylinder of:

$$\Delta F = A \cdot \Delta p = 9,45 \text{ cm}^2 \cdot 64,2 \frac{\text{daN}}{\text{cm}^2} = 6067 \text{ N.}$$

i.e. with a signal of 0.073 V at the servo valve, the cylinder produces a force of > 6000 N with which the disturbance forces are compensated.